

Strong Induction

The induction hypothesis is now,

$$(P(0) \wedge P(1) \wedge \dots \wedge P(k)) \rightarrow P(k+1)$$

Example University of Illinois: Proof Recurrence Relation by Strong Induction

Let a_n be a sequence where $a_1 = 1$ and $a_2 = 8$ and $a_n = a_{n-1} + 2a_{n-2}$. We want to prove that,

$$a_n = 3 \cdot 2^{n-1} + 2(-1)^n$$

We prove by induction on n . Initial case: $n = 3$

$$a_3 = 10$$

□

Inductive Step: $3 \leq n \leq k \rightarrow a_n = 3 \cdot 2^{n-1} + 2(-1)^n$

$$\begin{aligned} a_{n+1} &= a_n + 2a_{n-1} \\ &= 3 \cdot 2^{n-1} + 2(-1)^n + 2(3 \cdot 2^{n-2} + 2(-1)^{n-1}) \\ &= 2(3 \cdot 2^{n-1}) + 2(-1)^n + 2^2(-1)^{n-1} \\ &= 3 \cdot 2^n + 2(-1)^{n-1}(-1 + 2) \\ &= 3 \cdot 2^n + 2(-1)^{n-1} \\ &= 3 \cdot 2^n + 2(-1)^{n+1} \end{aligned}$$

□

We can use the induction hypothesis twice