Infinite Descent

Example Wikipedia: Proof by Infinite Descent

The square root of a non-integer is always irrational, formally,

$$k \notin \mathbb{N} \to \sqrt{k} \notin \mathbb{R} - \mathbb{Q}$$

Let $\sqrt{k}$ be a rational number and q the last integer before $\sqrt{k}$,

$$\sqrt{k} \in \mathbb{Q} \leftrightarrow \sqrt{k} = \frac{m}{n},\ m, n \in \mathbb{N}$$
$$q \in \mathbb{N} \wedge q < \sqrt{k} \wedge q + 1 > \sqrt{k}$$

We start by describing k,

$$\begin{aligned} \sqrt{k} &= \frac{m}{n} \\ &= \frac{m(\sqrt{k} - q)}{n(\sqrt{k} - q)} \\ &= \frac{m\sqrt{k} - mq}{n\sqrt{k} - nq} && (*2) \\ &= \frac{n\sqrt{k}\sqrt{k} - mq}{n\frac{m}{n} - nq} \\ &= \frac{nk - mq}{m - nq} \\ &\square \end{aligned}$$

Since there is an irreducible fraction for every rational number then the last expression is a contraction.

(*2) We want to get rid of $\sqrt{k}$ so we have to try to replace either the square itself or the term multiplying by the square root.