In[1]:=
```
mat = {{1, x1, y1, x1 * y1}, {1, x1, y2, x1 * y2}, {1, x2, y1, x2 * y1}, {1, x2, y2, x2 * y2}}
```

Out[1]=
$$\{\{1, x1, y1, x1\,y1\}, \{1, x1, y2, x1\,y2\}, \{1, x2, y1, x2\,y1\}, \{1, x2, y2, x2\,y2\}\}$$

In[2]:=
```
invmat = Simplify[Inverse @ mat]
```

Out[2]=
$$\left\{\left\{\frac{x2\,y2}{(x1-x2)(y1-y2)}, -\frac{x2\,y1}{(x1-x2)(y1-y2)}, -\frac{x1\,y2}{(x1-x2)(y1-y2)}, \frac{x1\,y1}{(x1-x2)(y1-y2)}\right\},\right.$$
$$\left\{-\frac{y2}{(x1-x2)(y1-y2)}, \frac{y1}{(x1-x2)(y1-y2)}, \frac{y2}{(x1-x2)(y1-y2)}, -\frac{y1}{(x1-x2)(y1-y2)}\right\},$$
$$\left\{-\frac{x2}{(x1-x2)(y1-y2)}, \frac{x2}{(x1-x2)(y1-y2)}, \frac{x1}{(x1-x2)(y1-y2)}, -\frac{x1}{(x1-x2)(y1-y2)}\right\},$$
$$\left.\left\{\frac{1}{(x1-x2)(y1-y2)}, -\frac{1}{(x1-x2)(y1-y2)}, -\frac{1}{(x1-x2)(y1-y2)}, \frac{1}{(x1-x2)(y1-y2)}\right\}\right\}$$

In[3]:=
```
vec = {{Q11}, {Q12}, {Q21}, {Q22}}
```

Out[3]=
$$\{\{Q11\}, \{Q12\}, \{Q21\}, \{Q22\}\}$$

In[4]:=
```
coefs = invmat.vec
```

Out[4]=
$$\left\{\left\{\frac{Q22\,x1\,y1}{(x1-x2)(y1-y2)} - \frac{Q12\,x2\,y1}{(x1-x2)(y1-y2)} - \frac{Q21\,x1\,y2}{(x1-x2)(y1-y2)} + \frac{Q11\,x2\,y2}{(x1-x2)(y1-y2)}\right\},\right.$$
$$\left\{\frac{Q12\,y1}{(x1-x2)(y1-y2)} - \frac{Q22\,y1}{(x1-x2)(y1-y2)} - \frac{Q11\,y2}{(x1-x2)(y1-y2)} + \frac{Q21\,y2}{(x1-x2)(y1-y2)}\right\},$$
$$\left\{\frac{Q21\,x1}{(x1-x2)(y1-y2)} - \frac{Q22\,x1}{(x1-x2)(y1-y2)} - \frac{Q11\,x2}{(x1-x2)(y1-y2)} + \frac{Q12\,x2}{(x1-x2)(y1-y2)}\right\},$$
$$\left.\left\{\frac{Q11}{(x1-x2)(y1-y2)} - \frac{Q12}{(x1-x2)(y1-y2)} - \frac{Q21}{(x1-x2)(y1-y2)} + \frac{Q22}{(x1-x2)(y1-y2)}\right\}\right\}$$

In[5]:=
```
x1 = x - 1; y1 = y - 1; x2 = x + 1; y2 = y + 1
```

Out[5]=
$$1 + y$$

In[6]:=
```
coefs
```

Out[6]=
$$\left\{\left\{\frac{1}{4}Q22\,(-1+x)(-1+y) - \frac{1}{4}Q12\,(1+x)(-1+y) - \frac{1}{4}Q21\,(-1+x)(1+y) + \frac{1}{4}Q11\,(1+x)(1+y)\right\},\right.$$
$$\left\{\frac{1}{4}Q12\,(-1+y) - \frac{1}{4}Q22\,(-1+y) - \frac{1}{4}Q11\,(1+y) + \frac{1}{4}Q21\,(1+y)\right\},$$
$$\left.\left\{\frac{1}{4}Q21\,(-1+x) - \frac{1}{4}Q22\,(-1+x) - \frac{1}{4}Q11\,(1+x) + \frac{1}{4}Q12\,(1+x)\right\}, \left\{\frac{Q11}{4} - \frac{Q12}{4} - \frac{Q21}{4} + \frac{Q22}{4}\right\}\right\}$$

In[7]:=
```
Simplify[coefs]
```

Out[7]=
$$\left\{\left\{\frac{1}{4}\left(-Q12\,(1+x)\,(-1+y)+Q11\,(1+x)\,(1+y)-(-1+x)\,(Q21+Q22+Q21\,y-Q22\,y)\right)\right\},\right.$$
$$\left\{\frac{1}{4}\left(Q21+Q22+Q12\,(-1+y)+Q21\,y-Q22\,y-Q11\,(1+y)\right)\right\},$$
$$\left.\left\{\frac{1}{4}\left((Q21-Q22)\,(-1+x)-Q11\,(1+x)+Q12\,(1+x)\right)\right\},\left\{\frac{1}{4}\left(Q11-Q12-Q21+Q22\right)\right\}\right\}$$

In[8]:=
```
Simplify[{1, x, y, x y}.Flatten[coefs]]
```

Out[8]=
$$\frac{1}{4}\left(Q11+Q12+Q21+Q22\right)$$

**k**